

1. An incompressible fluid of viscosity  $\mu$  is filled between two vertical coaxial cylinders as shown in Fig 1. The outer cylinder is rotating with an angular velocity  $\Omega_0$  and the inner one is kept stationary. If the end effects may be neglected, please determine the following terms assuming the fluid is under steady tangential laminar flow.

- (a) velocity distribution between the two cylinders (12 points)
- (b) shear stress distribution between the two cylinders (5 points)
- (c) the torque required to turn the outer cylinder (4 points)

The wetted surfaces of the outer and inner cylinders are kept at temperatures  $T_1$  and  $T_2$ , respectively. The fluid viscosity ( $\mu$ ), density( $\rho$ ), and thermal conductivity( $k$ ) are constants. If the value of  $\lambda$  is fairly small, please determine the following terms.

- (d) the temperature distribution between the two cylinders (10 points)
- (e) the maximum temperature within the shell (4 points)

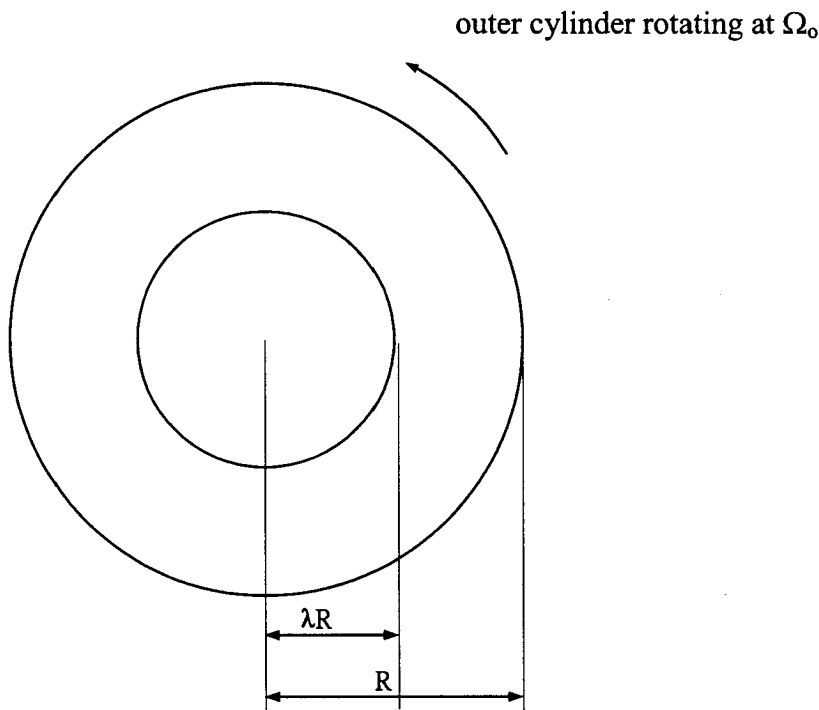


Fig 1

2. Please define the following terms and their physical interpretations.

- (a) Reynolds number (3 points)
- (b) Prandtl number (3 points)
- (c) Nusselt number (3 points)
- (d) Sherwood number (3 points)
- (e) Froude number (3 points)

3. A gas bubble is formed at the tip of a small tube that is completely immersed in a large body of liquid. The bubble remains in contact with the open end of the tube a way that allows us to control the pressure  $P$  inside the bubble. The bubble is small, the effect of gravity on the shape of the bubble is negligible, and therefore buoyancy forces will not lift the bubble from the tip of the tube. See Figure 2.

Develop a model (the governing equations) for the relationship of the rate of growth of the bubble to the pressure  $P$  and the properties of the fluid. List the necessary boundary conditions and all the assumptions you used to simply the model. (30 points)

4. A vertical surface is contacted with a continuous flow of liquid B flowing downward under the action of gravity (Figure 3). Gas A outside the liquid film is slightly soluble in B, but the viscosity of liquid B is not changed appreciably. Gas A absorbs by the laminar falling film and penetrates (diffuses) into the bulk phase of B. The penetration distance is small in comparison with the film thickness. Neglect the end effect and develop the governing equation for the diffusion of A into the falling film B. List the necessary boundary conditions and all the assumptions you used. You do not have to solve the differential equation. (20 points)

Figure 2. Gas bubble in a viscous liquid

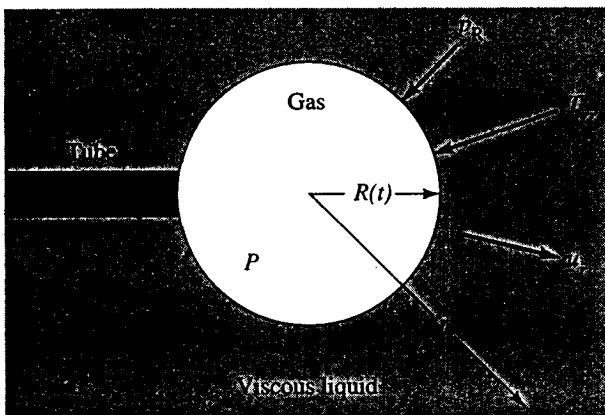
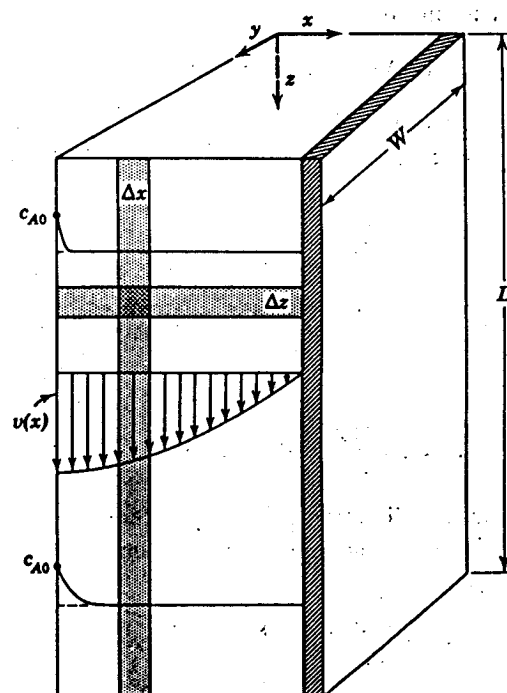


Figure 3 Absorption into a falling film



THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES  
(for Newtonian fluids of constant  $\rho$  and  $k$ )

*Rectangular coordinates:*

$$\begin{aligned} \rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) &= k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \\ &+ 2\mu \left\{ \left( \frac{\partial v_x}{\partial x} \right)^2 + \left( \frac{\partial v_y}{\partial y} \right)^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 \right\} + \mu \left\{ \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 \right. \\ &\left. + \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)^2 + \left( \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 \right\} \end{aligned} \quad (A)$$

*Cylindrical coordinates:*

$$\begin{aligned} \rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) &= k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] \\ &+ 2\mu \left\{ \left( \frac{\partial v_r}{\partial r} \right)^2 + \left[ \frac{1}{r} \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) \right]^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 \right\} + \mu \left\{ \left( \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right)^2 \right. \\ &\left. + \left( \frac{\partial v_r}{\partial r} + \frac{\partial v_r}{\partial z} \right)^2 + \left[ \frac{1}{r} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) \right]^2 \right\} \end{aligned} \quad (B)$$

*Spherical coordinates:*

$$\begin{aligned} \rho \hat{C}_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) &= k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \right. \\ &+ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \left. \right] + 2\mu \left\{ \left( \frac{\partial v_r}{\partial r} \right)^2 \right. \\ &+ \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left( \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right)^2 \left. \right\} \\ &+ \mu \left\{ \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]^2 + \left[ \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) \right]^2 \right. \\ &\left. + \left[ \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left( \frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right]^2 \right\} \end{aligned} \quad (C)$$

THE EQUATION OF CONTINUITY IN SEVERAL  
COORDINATE SYSTEMS

*Rectangular coordinates (x, y, z):*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

*Cylindrical coordinates (r,  $\theta$ , z):*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

*Spherical coordinates (r,  $\theta$ ,  $\phi$ ):*

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (\rho v_\phi) = 0$$

In terms of velocity gradients for a Newtonian fluid with constant  $\rho$  and  $\mu$ :<sup>a</sup>

$$\begin{aligned} r\text{-component} \quad & \rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \right) \\ & = -\frac{\partial p}{\partial r} + \mu \left( \nabla^2 v_r - \frac{2}{r^2} v_r - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{2}{r^2} v_\theta \cot \theta \right. \\ & \quad \left. - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_r \end{aligned} \quad (D)$$

$$\begin{aligned} \theta\text{-component} \quad & \rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2 \cot \theta}{r} \right) \\ & = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left( \nabla^2 v_\theta + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \right) + \rho g_\theta \end{aligned} \quad (E)$$

$$\begin{aligned} \phi\text{-component} \quad & \rho \left( \frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r} \cot \theta \right) \\ & = -\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi} + \mu \left( \nabla^2 v_\phi - \frac{v_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} \right. \\ & \quad \left. + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} \right) + \rho g_\phi \end{aligned} \quad (F)$$

<sup>a</sup> In these equations

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left( \frac{\partial^2}{\partial \phi^2} \right)$$

#### COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS IN CYLINDRICAL COORDINATES $(r, \theta, z)$

$$\tau_{rr} = -\mu \left[ 2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot v) \right] \quad (A)$$

$$\tau_{\theta\theta} = -\mu \left[ 2 \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) - \frac{2}{3} (\nabla \cdot v) \right] \quad (B)$$

$$\tau_{zz} = -\mu \left[ 2 \frac{\partial v_z}{\partial z} - \frac{2}{3} (\nabla \cdot v) \right] \quad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (D)$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu \left[ \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right] \quad (E)$$

$$\tau_{rz} = \tau_{zr} = -\mu \left[ \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right] \quad (F)$$

$$(\nabla \cdot v) = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \quad (G)$$