Transport Phenomena

Make assumptions and show all your work.

1. A viscous liquid is supplied from a vertical capillary tube to a solid cylinder of radius R, and the resulting film of liquid flows radially across the top of cylinder to the outer edge. Beyond the edge, the liquid flows down the outside of the cylinder under gravity.

(a) (25%) If the film thickness, δ, along the outside of the cylinder is constant, derive an expression for the velocity profile $u_z(r)$ for the film flow in the region along the side of the cylinder. State all the assumptions you use.

(b) (25%) The temperature of the cylinder is held constant at T_w, and the ambient temperature outside the falling film is T_o with a convection transfer coefficient h. If heat dissipation of fluid viscosity CANNOT be neglected, derive temperature distribution within the falling film.

2. (15%) Write down
 (a) the equation of continuity in index notion (or the general form).
 (b) the Navior-Stokes equation.
 (c) the assumptions for the Navior-Stokes equation.

3. (15%) A fluid (with depth H) is above a plate and is induced at t = 0 by the bottom plate that oscillates sinusoidally [V cos(at)].
 (a) Write down the governing equation (GE) and simply the GE by making some assumptions.
 (b) Write down the initial and boundary conditions.
 You do not have to solve the GE.

4. (20%) Consider the mass transfer of a solute A into a laminar falling film (B), as shown in the Figure. Solute A is only slightly soluble in B and the diffusion take place so slowly in the liquid film B that A will not penetrate very far into B.

(a) Establish a mass balance on component A and obtain the governing equation (GE).

(b) Simplify the GE (detail the reasons) to result GE:

$$v_z \frac{\partial c_A}{\partial z} = D_{AB} \frac{\partial^2 c_A}{\partial x^2}.$$

(c) write down the boundary conditions, and explain the physical meaning also, for solving the GE.

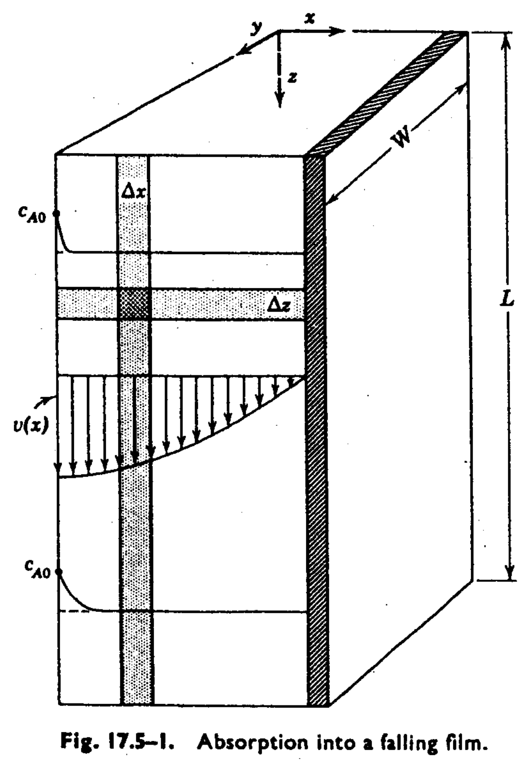

Fig. 17.5-1. Absorption into a falling film.

TABLE 10.2-3

THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES

(*for Newtonian fluids of constant ρ and k*)

(Eq. 10.1-25 with viscous dissipation terms included)

Rectangular coordinates:

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

$$+ 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2\right.$$

$$\left. + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \qquad (A)$$

Cylindrical coordinates:

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

$$+ 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left[\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\right]^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2\right.$$

$$\left. + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)^2 + \left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]^2\right\} \qquad (B)$$

Spherical coordinates:

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right.$$

$$\left. + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2\right.$$

$$\left. + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2\right\}$$

$$+ \mu\left\{\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2\right.$$

$$\left. + \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2\right\} \qquad (C)$$

Note: The terms contained in braces { } are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.